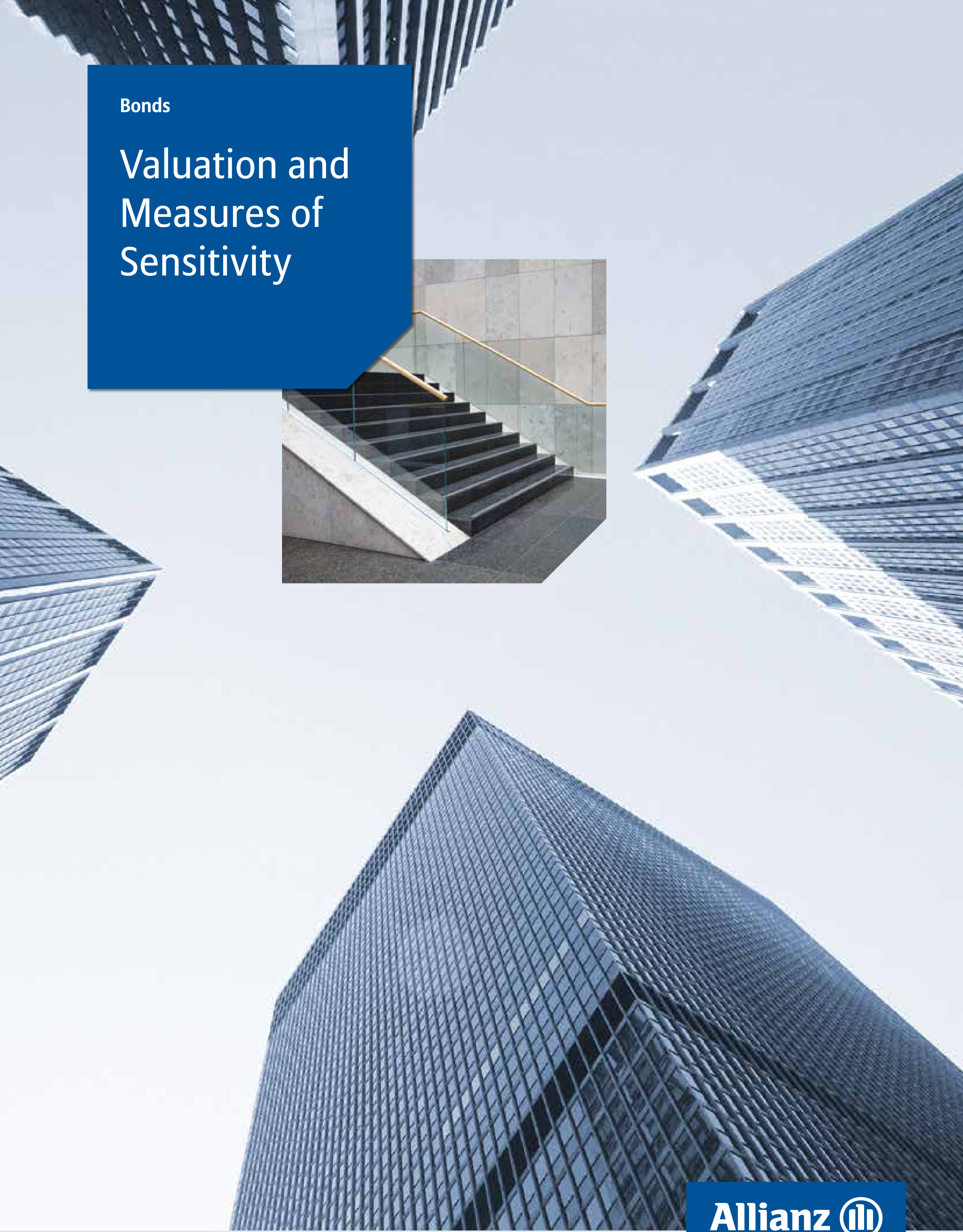

Bonds

Valuation and Measures of Sensitivity

Allianz Global Investors

Understand. Act.

Maturity management is essential, especially if a risk profile is asymmetric, as is typically the case when interest rates are low.

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Valuing and measuring the sensitivity of bonds as a basis for maturity management

Prices of (government) bonds are also subject to fluctuation. This is nothing new, but something that can be put to specific good use, e. g. by backing maturities.

Bond markets are more interesting than they might seem at first glance. Anyone wanting to manage a bond portfolio must understand the market's most important indicators. "Duration" and "convexity" are two key words that come to mind. If you know what they mean, you can minimize interest rate risks, hedge prices or bet directly on rising prices.

We are going to take a closer look at bond valuation and price sensitivity below, and examine how practicable they are for investors. A second part focuses on hedging and speculation strategies.

Price

Let us start by looking at bond prices: Once we understand how bond prices are formed, we can use this knowledge to derive all other effects. Our rationale uses government bonds from the industrial countries as our examples since they are often used as benchmarks.

The "present value method" serves as the basis: the present value of a bond equates to the value of future cash flows discounted to the present (see Box 2).

Box 1: Explanatory notes on the yield curve

Par

Above par: Bond price $K > 100$

Par: $K = 100$

Below par: $K < 100$

Yield curve

The yield curve represents the internal rates of return of a zero-coupon bond relative to the duration of investment in zero-coupon bonds with the same rating.

Put simply: It assigns a matched-maturity interest rate to each zero-coupon bond at which the zero-coupon bond is discounted to the present value at the starting point. In practice, the corresponding yields of government bonds across the maturity spectrum are used.

A distinction is made between three types of increase in the yield curve:

Normal: Interest rates increase along with maturities.

Flat: Interest rates stay the same regardless of maturity.

Inverted: Interest rates decrease as residual maturity increases.

We know:

- The redemption price K (also known as the nominal value), which is generally 100;
- The residual maturity T of the bond;
- The coupon c and payment intervals (in this example: annual payment);
- The interest rates i that differ according to payment date and are derived from the yield curve, based on maturity t (see Box 1).

We are looking for the price of the bond.

Example

The price of a bond with a redemption price of 100 and a residual maturity of five years is currently 100.50 (as an example), arrived at by discounting the individual payments (coupons and redemption value) at matched-maturity rates to the time of valuation (see Chart 1). Specifically, this means that a coupon that is due for payment in one year is discounted at the rate applicable for one year, a coupon due in two years at the rate applicable for two-year maturities, etc.

Box 2: Calculating present value

The current price equates to the aggregate payments of the future, discounted at the relevant matched-maturity interest rate.

Zero-coupon bonds are a special case, where the coupon c is correspondingly set to zero.

$$K_0 = \sum_{t=1}^{T} \frac{c}{(1+i_t)^t} + \frac{100}{(1+i_T)^T}$$

Price = what is paid today for the cash flows of the future.

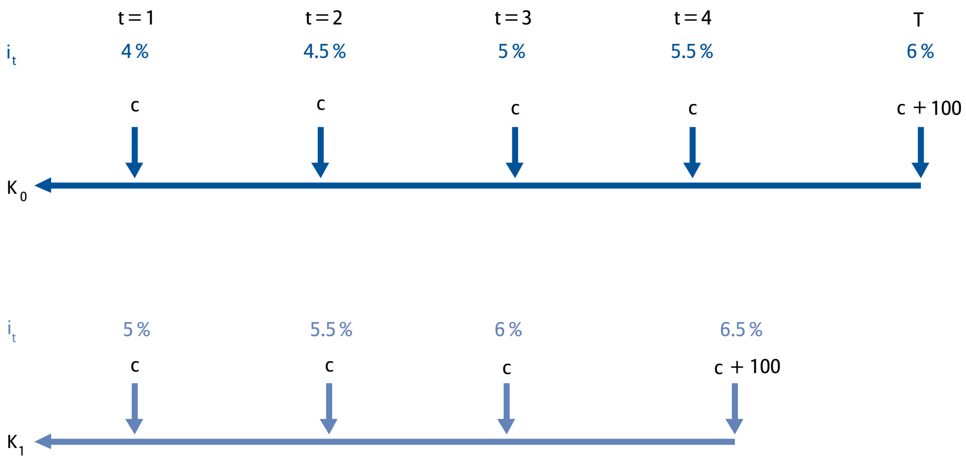
At the end of one year, the investor looks at the portfolio and sees that the bond is worth 102.01. Money from heaven?

Does money fall from heaven?

Example 1: time effect

Unfortunately, no. This is the time effect. The initial five-year bond has become a four-year bond. If the yield curve is "normal", the discounting factors decrease over time. In the case of a bond that started above par, prices initially rise before moving closer to the redemption price as final maturity approaches. Experts call this the "rolling down the yield curve effect".

Chart 1: Discounting principle



In this example: Maturity T decreases by one year. Where t = 4, the bond has a residual maturity of four years. At the same time, interest i along the yield curve has risen by half a percentage point in each case.

Source: Allianz Global Investors Global Capital Markets & Thematic Research

Example 2: Interest rate effect

Bonds are subject to interest rate risk.

Same bond, same initial price. But after one year the bond is only worth 98.44. What has happened?

The roll-down effect on its own would have pushed the bond price up to 102.01. In the meantime, however, the yield curve has shifted upwards, meaning that interest rates have risen over the entire range of maturities. Whereas the coupon that was initially due in two years would have been discounted at 4% instead of 4.5% (time effect), it is now discounted at the new rate of 5% for one-year maturities following the general rise in interest rates. The discount factor has increased. The risk event of a change in interest rates has occurred.

When (expected) changes in interest rates occur, the question we must always ask ourselves is how much will the bond price rise/fall if the yield falls/rises?

Interest rate risk only arises if bonds are not held until final maturity. On the other hand, it can also present an opportunity: When interest rates dip in the capital markets, prices rise temporarily.

But how strongly will these impacts affect prices – for good or for bad? What risks is an investor taking, what opportunities may arise? It's all about the sensitivity with which bonds react to changes in interest rates. Duration is a key measure of the price sensitivity of bonds.

Box 3: Calculating Macaulay Duration

Duration is calculated by multiplying coupons and the redemption amount with the relevant holding period and discounting them to the present:

$$D = (1/K) \sum_{t=1}^{T} t \times CF_t/(1+i)^t$$

In the case of Macaulay duration, the payments are discounted at the internal rate of return, which is the yield to maturity.

i = Internal rate of return (yield to maturity)

K0 = Price of initial investment

CFt = Cash flow at point in time t

Duration

Various definitions of duration exist. The Macaulay duration that is used here is the most common definition (see Box 3).

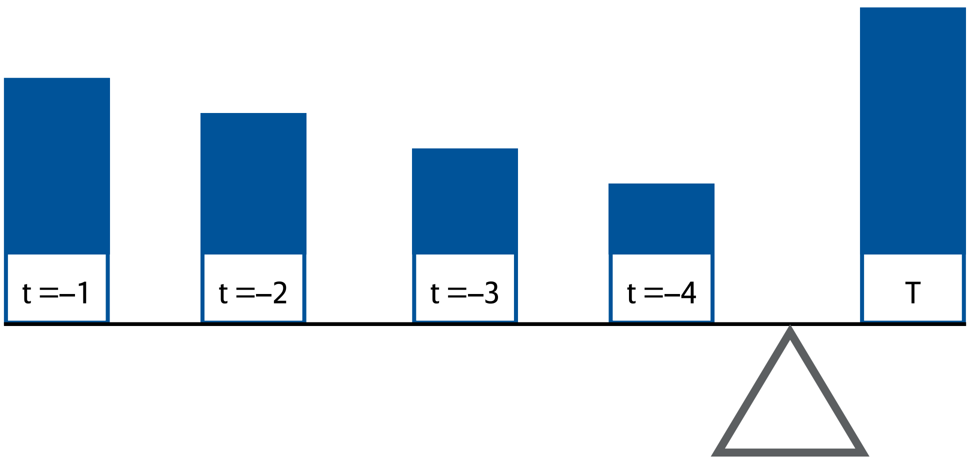
Conceptually, two interpretations with similar content exist for duration. It can mean

- the weighted commitment period of the cash flows of a bond, whereby the commitment period of each payment is weighted with its share in the present value of the bond;

Chart 2: Duration – the basic concept

Duration is like a pair of scales, keeping the cash flow level.

Future payments lose weight as discounting increases. The redemption price is due at point in time T.



Source: Allianz Global Investors Global Capital Markets & Thematic Research

- the average maturity of a bond, taking account of all cash flows, i. e. coupon payments and redemption amount.

Put simply: If a bond with a maturity of five years has a duration of four years, the initial capital is committed for four years on average.

Based on duration, the price sensitivity with which a bond reacts to changes in yield can be derived through simple conversion. Modified duration is generally used for this purpose (see Box 4). If the modified duration is four, the price of a bond will drop 4 % if interest rates rise by 1 %.

(Modified) duration can be applied to an entire bond portfolio, such as a bond fund. It answers the question of how strongly the value of my bond portfolio will rise/fall if interest rates rise/fall by a certain amount.

Characteristics of duration

Duration is like a pair of scales, keeping cash flows in balance (see Chart 2). It is exactly equal to the balance point between the individual coupon payments and the redemption price. This diagram visualizing duration as a measure of the weighted commitment period of the invested capital illustrates its characteristics clearly:

- Since cash flows are discounted to the present value at the time of calculation, the following applies: The earlier payments are scheduled, the lower their discount factor and the greater their weighting.
- The smaller the coupon, the greater the duration since the redemption price gains weight in relative terms. If we stay with our visualization example, then the balance point moves further to the right. In the special case of zero-coupon bonds, the maturity is exactly equivalent to the duration.

Box 4: Duration and price movement

Price = Present value of the cash flows:

$$K = \sum_{t=1}^{T} CF_t/(1+i)^t$$

derived according to i:

$$dK/di = -\sum_{t=1}^{T} t \times CF^t/(1+i)_{(t+1)}$$

which produces:

$$dK/di = -D \times P/(1+i)$$

or, equally:

$$dK/K = -D/(1+i)\ di$$

or:

$$dK/K = -DM\ di$$

If the yield i rises by 1 %, the price K falls by DM %.

DM stands for “modified duration”, which is commonly used.

- Bonds do not have a constant duration. On the contrary, their duration is dictated by market conditions including coupon amount, residual maturity and (present) yield curve. This is an important point, especially with regard to hedging strategies, as price hedges always have to be adjusted to keep pace with changes in duration.
- As interest rate levels rise on the capital markets, duration decreases since future cash flows are discounted more strongly.
- Duration decreases linearly over time between coupon dates. Duration jumps around a coupon date. The old balance point shifts to the right.

Duration is not constant.

Convexity

However: Duration is merely an approximate measure for minor changes in interest rates. It becomes an inaccurate measure of price changes when interest rates change more dramatically because it assumes a linear relationship between a change in the interest rate and a change in price. This relationship is, however, actually convex. As such, duration leads to inaccuracies that increase in magnitude the greater the change in interest rates (see Chart 3).

If interest rates fall, duration dictates a change in price that is smaller than the actual change. If interest rates rise, duration produces a bigger change in price than is actually the case. Convexity is the exact measure of price sensitivity (see Box 5).

Chart 3 clearly illustrates the properties of convexity:

- As the (internal) rate of return increases, the convexity of the bond decreases in line with the gradient of the curve.
- For a given yield and residual term, convexity increases as the coupon decreases. Accordingly, a zero-coupon bond reacts most strongly to a change in yield.

Box 5: Calculating convexity

The convexity of a bond is derived by approximating a square root using a Taylor series. Producing:

$$\text{Konv} = \left(\sum_{t=1} CF_t/(1+i)^t * t * (t+1)\right)/K$$

For exact calculation, see "Bond Markets, Analysis and Strategies", Frank J. Fabozzi, 2000
Frank J. Fabozzi, 2000.

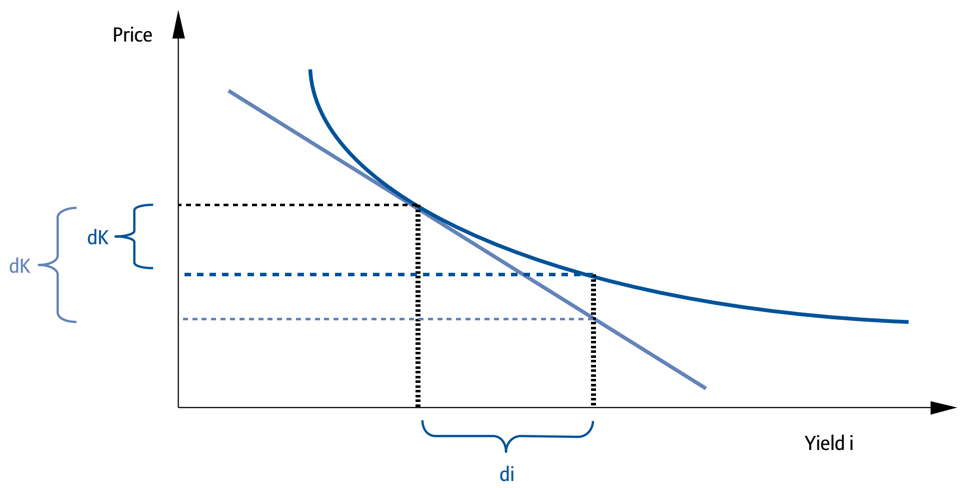
- As duration increases, the convexity of a bond grows at an increasing rate. So if an investor swaps one bond for another one with twice the duration, the convexity more than doubles. Which also means that the measurement accuracy of duration increases compared with convexity measurement.

Maturity management is essential, especially if a risk profile is asymmetric, as is typically the case when interest rates are low.

It may make sense to bet specifically on maturities that suit your own investment horizon in order to limit the duration risk and yet still earn a better return than can be found on the money market.

Chart 3: Duration vs. Convexity

Duration is merely an approximation. Convexity is more accurate.



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